Alpha, Beta, and Now…Gamma

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Abstract

When it comes to generating retirement income, investors arguably spend the most time and effort on selecting 'good' investment funds/managers—the so called alpha decision—as well as the asset allocation, or beta, decision. However, alpha and beta are just two elements of a myriad of important financial planning decisions, many of which can have a far more significant impact on retirement income.

We introduce a new concept called "Gamma" designed to quantify the additional expected retirement income achieved by an individual investor from making more intelligent financial planning decisions. Gamma will vary for different types of investors, but in this article we focus on five fundamental financial planning decisions/techniques: a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products (i.e., annuities), tax-efficient decisions, and liability-relative asset allocation optimization.

We estimate a retiree can expect to generate 29% more income on a "utility-adjusted" basis using a Gamma-efficient retirement income strategy when compared to our base scenario, which assumes a 4% constant real withdrawal and a 20% equity allocation portfolio. This additional income is equivalent to an annual arithmetic return increase of +1.82% (i.e., Gamma equivalent alpha), which represents a significant improvement in portfolio efficiency for a retiree. Unlike traditional alpha, which can be hard to predict, we find that Gamma (and Gamma equivalent alpha) can be achieved by anyone following an efficient financial planning strategy.

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The potential benefits from "good" financial planning decisions are often difficult to quantify. For any given portfolio, investment decisions can generally be decomposed into two primary components: beta and alpha. Beta can generally be defined as the systematic risk exposures of the portfolio (usually achieved through asset allocation), while alpha is the residual, or skill/luck-based, component associated with the various flavors of active management (e.g. tactical asset allocation, security selection, etc.). Alpha and beta are at the heart of traditional performance analysis; however, as we show in this article, their impact on a successful retirement can be far less important than other financial planning decisions.

In this article we introduce a new concept called "Gamma" designed to measure the additional expected retirement income achieved by an individual investor from making more intelligent financial planning decisions. Gamma is the third letter in the Greek alphabet (preceded by alpha and beta) and within in financial economics is sometimes used as the variable denoting the investor's degree of risk aversion. Given that Gamma is relatively unclaimed within the financial literature, we seek to give it a new meaning. Gamma varies for different investors as well as for investors in different lifecycles (e.g., accumulation versus retirement). For those that find it hard to break from traditional (and inadequate) performance measurement, Gamma is a metric that is somewhat comparable to an alpha, or excess return, but it is the return an investor experiences based on optimal financial decision making.

We focus on five important financial planning decisions/techniques: a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products (i.e., annuities), tax-efficient allocation decisions, and a portfolio optimization that includes the liability. Each of these five Gamma components creates value for retirees, and when combined, can be expected to generate 29% more income on a utility-adjusted basis when compared to a simplistic static withdrawal strategy according to our analysis. This additional income is equivalent to an arithmetic "alpha" of 1.82% (i.e., Gamma equivalent alpha) and thereby represents a significant potential increase in portfolio efficiency (and retirement income) for retirees.

Alpha and Beta: Defining Value

The notions of beta and alpha (in particular alpha) have long fascinated financial advisors and their clients. "Alpha" allows a financial advisor to demonstrate (and potentially quantify) the excess returns generated, which can help justify fees. In contrast, beta (systematic risk exposures) helps explain the risk factors of a portfolio to the market, i.e., the asset allocation.

Quantifying Beta

The importance of the asset allocation decision (the beta decision) has been one of the most controversial and emotional subjects of the past 25 years. The firestorm began with Brinson, Hood, and Beebower

(1986), which finds that the variance of a portfolio's asset allocation, or policy return, explained 93.6% of the variation in the 91 large U.S. pension plans tested. Brinson, Singer, and Beebower (1996) confirm the results in the original paper, but found a slightly lower number, 91.5%. While the results of the Brinson studies became an accepted and often misinterpreted "truth," other researchers were more circumspect. In an important but little noticed paper, Hensel, Ezra, and Ilkiw (1991), points out that a naïve portfolio must be chosen as a baseline in order to evaluate the importance of asset allocation policy. They point out that in the Brinson studies the baseline portfolio is 100% cash so that these studies are demonstrating the self-evident fact that investing in risky assets produces volatile returns. Janke (1997) caused a great deal of debate with an article titled "The Asset Allocation Hoax."

In our view, the debate was nearly settled by Ibbotson and Kaplan (2000), which concluded that "while asset allocation explains about 90% of the variability of a fund's returns over time, it explains only about 40% of the variation of returns across funds." The settling of the debate and the proper interpretation of the "40% of the variation of returns across funds" was finally provided by Xiong, Ibbotson, Idzorek, and Chen (2010), who found that after controlling for interaction effects, about three-quarters of a typical fund's variation in time-series returns comes from general market movement, with the remaining portion split roughly evenly between the specific asset allocation and active management. For an excellent summary of the asset allocation debate, we recommend Ibbotson (2010) and Idzorek (2010).

Quantifying Alpha

The concept of alpha is far more difficult to quantify. Sharpe (1992) concludes that style and size explain 80%-90% of mutual fund returns, while stock selection explains only 10%-20%. There have been numerous active versus passive studies, the majority of which suggest that alpha (when correctly measured) likely does not exist after taking fees into account. Therefore, if a financial advisor's value proposition is focused on the notion of "adding alpha" and he or she is not able to generate alpha (which should hold in aggregate), has the advisor still added value? The answer to this question depends on a variety of factors, but primarily the scope of the relationship with the client.

Beyond Beta and Alpha

If an advisor is paid solely to manage a portfolio of assets, and does nothing else, i.e., offers no additional advice regarding anything other than the investment of the client assets, the concepts of alpha and beta should be relatively good measures of the value of the advisor. However, in more complex engagements, in particular as it relates when providing financial planning services to clients, value cannot be defined in such simple returns as alpha and beta, since the objective of an individual investor is typically to achieve a goal, and that goal is most likely saving for retirement.

It may be that a financial advisor generates significant negative alpha for a client (i.e., invests the client's money in very expensive mutual funds that underperform), but still provides other valuable services that enable a client to achieve his or her goals. While this financial advisor may have failed from a pure alpha perspective, the underlying goal was accomplished. This is akin to losing a battle but winning the war.

Individual investors invest to achieve goals (typically an inflation-adjusted standard of living), and doing the things that help an investor achieve those goals (i.e., adding Gamma) is a different type of value than can be attributed to alpha or beta alone, and is in many ways more valuable. Therefore, asset-only metrics are an incomplete means of measuring retirement strategy performance.

Gamma Factors

In this paper, we examine the potential value, or Gamma, that can be obtained from making "intelligent" financial planning decisions during retirement. A retiree faces a number of risks during retirement, some of which are unique to retirement planning and are not concerns during accumulation. We will explore five different Gamma factors:

1. Total Wealth Asset Allocation

Using human capital in conjunction with the market portfolio to determine the optimal equity allocation. Most techniques used to determine the asset allocation for a client are relatively subjective and focus primarily on risk preference (i.e., an investor's aversion to risk) and ignore risk capacity (i.e., an investor's ability to assume risk). In practice, however, we believe asset allocation should be based on a combination of risk preference and risk capacity, although primarily risk capacity. We determine an investor's risk capacity by evaluating his or her total wealth, which is a combination of human capital (an investor's future potential savings) and financial capital. We can then either use the market portfolio as the target aggregate asset allocation for each investor (as suggested by the Capital Asset Pricing Mode) or build an investor-specific asset allocation that incorporates an investor's risk preferences. In both approaches, the financial assets are invested, subject to certain constraints, in order to achieve an optimal asset allocation that takes both human and financial capital into account.

2. Dynamic Withdrawal Strategy

The majority of retirement research has focused on static withdrawal strategies where the annual withdrawal during retirement is based on the initial account balance at retirement, increased annually for inflation. For example, a "4% Withdrawal Rate" would really mean a retiree can take a 4% withdrawal of the initial portfolio value and continue withdrawing that amount each year, adjusted for inflation. If the initial portfolio value was $1 million and the withdrawal rate was 4%, the retiree would be expected to generate $40,000 in the first year. If inflation during the first year was 3%, the actual cash flow amount in year two (in nominal terms) would be $41,200. Under this approach, the withdraw amount is based entirely on the initial income target, and is not updated based on market performance or expected investor longevity. The approach we use in this paper, originally introduced by Blanchett, Kowara, and Chen (2012), determines the annual withdrawal amount annually based on the ongoing likelihood of portfolio survivability and mortality experience.

3. Annuity Allocation

Outliving one's savings is perhaps the greatest risk for retirees. For example, a study by Allianz Life noted that the greatest fear among retirees is not death (39%) but rather outliving one's resources (61%) (See Bhojwani [2011]). Annuities allow a retiree to hedge away this risk and can therefore improve the overall efficiency of a retiree's portfolio. The contribution of an annuity within a total portfolio framework, (benefit, risk, and cost) must be considered before determining the appropriate amount and annuity type.

4. Asset Location and Withdrawal Sourcing

Tax-efficient investing for a retiree can be thought of in terms of both "asset location" and intelligent withdrawal sequencing from accounts that differ in tax status. Asset location is typically defined as placing (or locating) assets in the most tax-efficient account type. For example, it generally makes sense to place less tax-efficient assets (i.e. those where the majority of total return comes from coupons/

dividends taxed as ordinary income), such as bonds, in retirement accounts (e.g., IRAs or 401ks) and more tax-efficient assets (i.e. those where the majority of total return comes from capital gains taxed a rate less than ordinary income), such as stocks, in taxable accounts. When thinking about withdrawal sequencing, it typically makes sense to withdrawal monies from taxable accounts first and more tax-efficient accounts (e.g., IRAs or 401ks) later.

5. Liability-Relative Optimization
Asset allocation methodologies commonly ignore the funding risks, like inflation and currency, associated with an investor's goals. By incorporating the liability into the portfolio optimization process it is possible to build portfolios that can better hedge the risks faced by a retiree. While these "liability-driven" portfolios may appear to be less efficient asset allocations when viewed from an asset-only perspective, we find they are actually more efficient when it comes to achieving a sustainable retirement income.

From a more holistic perspective, each of these Gamma concepts can be thought of as actions and services provided by financial planners. This is a concept Bennyhoff and Kinniry (2011) called "Advisor's alpha" and Scott (2012) calls "household alpha." However, Bennyhoff and Kinniry do not attempt to quantify the potential benefit of these actions and discuss the implications in a more qualitative fashion and Scott focuses solely on the potential benefit from optimal Social Security claiming decisions. However, he does note the potential use of a utility function to measure the tradeoffs involved in the Social Security decision. In this article, we take a utility-function approach to quantify the benefit of different income-maximizing decisions The goal of this article is to provide some perspective, as well as quantify, the potential benefits that can be realized by an investor (in particular a retiree) from using a Gamma-optimized portfolio.

Measuring Gamma

Gamma measures how much additional utility-adjusted income a strategy in question adds over and above the utility-adjusted income from a set of base-case decisions. For those readers not familiar with the concept of utility, it is an approach to quantify the satisfaction derived from some set of goods or services. In this case, we use a utility function to give greater weight to more certain outcomes and outcomes nearer to the present. For example, we assume that the utility of income is an increasing concave function so that the higher the level of income, the lower the increase in utility of additional income. So, the amount of utility an investor gets for each dollar of income is not equal.

To measure Gamma, we first calculate the utility-adjusted income generated by the Gamma-optimized portfolio, which we denote as II. We perform Monte Carlo simulation and use the results to calculate II for each sample path or trial. Specifically, we define II as the constant amount of income that a retiree would accept such that his or her utility would equal the utility of the actual income path realized on a given simulation path[1]. This is given by:

$$II = \left(\frac{\sum_{t=0}^{T} q_t (1+\rho)^{-t} I_t^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{T} q_t (1+\rho)^{-t}} \right)^{\frac{\eta}{\eta-1}} \qquad [1]$$

where:

I_t = the level of income in year t

q_t = the probability of surviving to at least year t

T = the last year for which qt>0

ρ = the investor's subjective discount rate

η = the investor's elasticity of intertemporal substitution preference parameter

Note that while equation [1] contains two preference parameters (ρ and η) that describe how the investor feels about having income to consume at different points in time, it makes no reference to how the investor feels about risk. Following the approach in Epstein and Zin (1989), we treat the elasticity of intertemporal substation as a parameter distinct from the risk tolerance parameter. We introduce the risk tolerance parameter next by treating the path as unknown and evaluating expected utility.

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[1] Williams and Finke (2011) use a similar concept to assess the relative attractiveness of different withdrawal rates.

To define expected utility, we introduce the risk tolerance parameter, $\boldsymbol{\theta}$. We define the expected utility of a strategy as:

$$EU = \sum_{i=1}^{M} p_i \frac{\theta}{\theta - 1} (II_i)^{\frac{\theta-1}{\theta}} \qquad [2]$$

where M is the number of paths, the subscript i to denote which of M paths is being referred to, and pi is the probability of path i occurring which we set to $1/M$.

We define Y as the constant value for II that we yield this level of expected utility. This is the certainty-equivalent of the stochastic utility-adjusted income II. Y is given by:

$$Y = \left[\sum_{i=1}^{M} p_i \, (II_i)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad [3]$$

We can now formally define the Gamma of a given strategy or set of decisions as:

$$Gamma(Strategy) = \frac{Y(Stategy) - Y(Benchmark)}{Y(Benchmark)} \qquad [4]$$

For the analysis below we use the following values $\boldsymbol{\rho}$ = 5%, $\boldsymbol{\eta}$ = 0.5, and $\boldsymbol{\theta}$ = 0.33.
A sensitivity analysis was performed and the results were relatively unaffected by the values of $\boldsymbol{\rho}$, $\boldsymbol{\eta}$, and $\boldsymbol{\theta}$.

Key Analysis Assumptions

In order to determine the impact on Gamma from the five different decisions, three entirely different "tests" were performed. The first test was used to calculate the contribution to Gamma from total wealth asset allocation, annuity allocation, and dynamic withdrawal strategy. This simulator was a modified version of that used by Blanchett, Kowara, and Chen (2012). The second test was to determine the impact of liability-relative optimization, and the final test was to determine the impact of asset location and withdrawal sourcing.

While ideally a single generator would have been used to quantify the unique contribution of each of the five types, the individual tests are by themselves considerably complex and the separation was done out of necessity. Therefore, in order to determine the aggregate Gamma from the three different tests the results must be combined. The increase in utility-adjusted income (i.e., Gamma) could be multiplicative, additive, or neither and is something we leave for future research. Here, for simplicity purposes, we assume the improved income that could be generated are additive across the three tests since each of the tests are relatively independent (i.e., each quantifying some different aspect of potential "financial planning alpha").

We calculate the survival probabilities from the mortality rates presented in Johnson (1998). Johnson presents mortality rates for both males and females that we average to create a unisex mortality table. Using these data we calculate survival probabilities for a 65 year old.

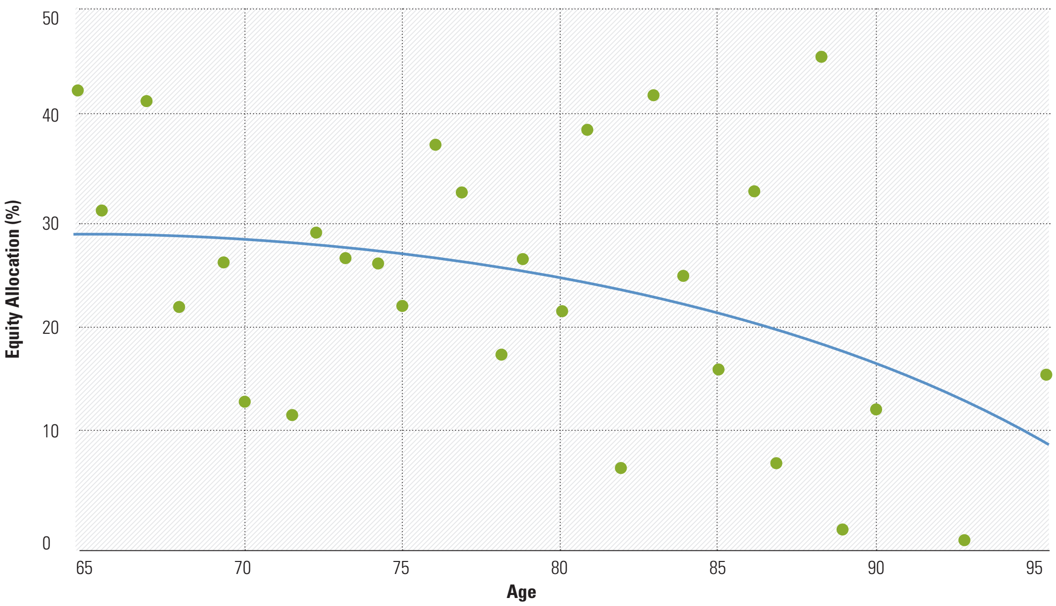
We present the return assumptions in the Appendix. The returns and standard deviations are based on Ibbotson's Capital Market Assumptions (CMAs) as of December 31, 2011. The correlations, skewness, and kurtosis values used to generate the multivariate non-normal distribution are based on annual calendar year returns for the respective asset classes from 1973 to 2011. Note, the respective correlations, skewness, and kurtosis values for US TIPS are only from 1998 to 2011 since US TIPS were not introduced until 1997. While synthetic proxies do exist for TIPS, the creators of these assumptions decided to solely use actual historical data due to the difficulties associated with accurately backfilling this complex asset class.

We performed a multivariate simulation of 10,000 returns for each asset class based on the values the Appendix. We used the Truncated Lévy Flight (TLF) distribution presented by Xiong and Idzorek (2011). The TLF distribution is a skewed fat-tailed distribution that reflects the statistical properties that are found in historical asset class return data as documented by Kaplan (2012) and Xiong (2010).

Base Case

In order to determine the potential benefit associated with more intelligent financial planning decisions for retirees (i.e., Gamma), we created a base case. The overall "intelligence" of the base scenario will obviously affect the potential gains available through more advanced approaches. We assume a relatively intelligent base scenario, where the retirees (a male and a female both age 65) would follow the "4% rule," which is based on the initial balance, where the actual dollar amount is based on the initial withdrawal increased annually by inflation. The base equity allocation is assumed to be 20%, which is the approximate average for investors from age 65 to 95 based on the 2010 Survey of Consumer Finances, as shown in Figure 1.

Figure 1: Equity Allocation by Age

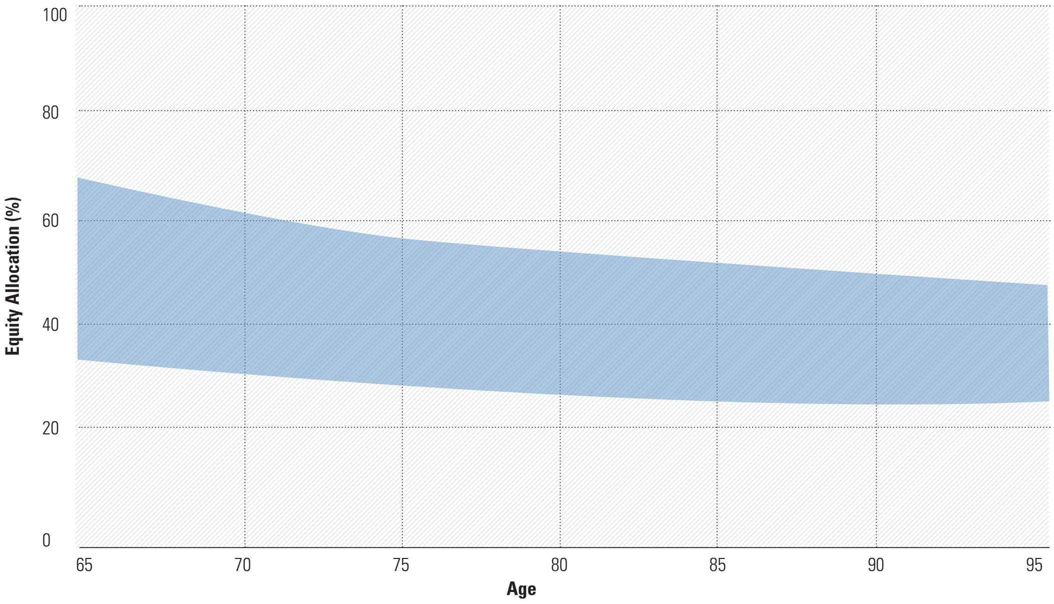


Source: 2010 Survey of Consumer Finances

The actual portfolios tested vary by simulations. For the first test, we assume a "naïve" portfolio (since the potential benefits of more complex optimizations is considered in the second test) where the 80% fixed-income portion of the portfolio is invested in 75% US Bonds and 25% in Cash and the 20% equity piece is invested in 50% US Large-Cap Stocks, 25% US Small-Cap Stocks, and 25% Non-US Large Cap Stocks. For the second test, we test more precise allocations based on different optimization methodologies. The third test uses returns on US Large-Cap Stocks and US Bonds.

For the total wealth asset allocation test, we place a boundary on the maximum and minimum potential equity allocation for the Gamma-optimized portfolio. We based the boundaries on the equity allocations of the Morningstar target-date indices (Aggressive and Conservative, respectively). These glide paths are included in Figure 2.

Figure 2: Morningstar Target-Date Index Equity Allocations and Total Wealth Asset Allocation Equity Boundaries



Source: Morningstar Inc and Ibbotson Associates

First Test: Total Wealth Asset Allocation, Annuity Allocation, and Dynamic Withdrawal Strategy

With the first test we are able to estimate the Gamma of total wealth asset allocation, annuity allocation, and dynamic withdrawal strategy. For the total wealth asset allocation test we assume the overall optimal portfolio has an equity allocation of 45% to match the market portfolio, including both public securities as well as non-publically traded instruments. We also assume that the mortality-weighted net present value of the annuity and/or Social Security income is "bond-like," i.e., 0% equity. Given this assumed allocation, the remaining financial assets are invested in order to achieve a target equity allocation of 45%, (the assumed equity allocation of the overall optimal portfolio). Note, though, the equity allocation is bounded between the high and low glide paths in Figure 2.

For the annuity allocation simulation, we assume that 25% of the total retirement assets are used to purchase a fixed immediate annuity that we assume has a payout rate of 5.71%. We obtained this rate from immediateannuities.com for a joint couple, male and female, both age 65, with 100% survivor benefit in July 2012.

The dynamic withdrawal strategy is based on the "Mortality Updating Failure Percentage" approach of Blanchett, Kowara, and Chen (2012) where the probability of failure parameter is 25% and the probability of outliving the distribution period parameter is also 25%. Under this approach, the percentage withdrawn from the portfolio will vary in a given year based on assumed remaining life expectancy of the retiree/s and the portfolio equity allocation. Table 1 includes a sample of these withdrawal rates. For example, if the portfolio value is $100,000, the equity allocation is 40%, and the remaining expected life expectancy is 20 years, the withdrawal amount would be $5,900 (which is 5.9% of the $100,000 portfolio).

Table 1: Dynamic Withdrawal Strategy Portfolio Withdrawal % by Equity Allocation and Number of Years Remaining

	Equity Allocation				
Number of Years Remaining	20%	30%	40%	50%	60%
5	20.0%	19.9%	19.9%	19.8%	19.9%
10	10.4%	10.4%	10.5%	10.4%	10.5%
15	7.2%	7.3%	7.4%	7.4%	7.5%
20	5.7%	5.8%	5.9%	6.0%	6.0%
25	4.8%	4.9%	5.0%	5.1%	5.2%
30	4.2%	4.4%	4.5%	4.6%	4.7%
35	3.8%	3.9%	4.1%	4.2%	4.3%
40	3.5%	3.6%	3.8%	3.9%	4.0%

Social Security income is assumed to be half of the total annual real income target of the joint couple and therefore represents an asset that is 50% of the total value of the assets held by the retiree. While the precise required minimum distributions (RMD) rules are not considered within the withdrawal process since it is based on remaining mortality, the annual distributions do approximately equal RMDs.

Since it is not possible to test for the impact of total wealth asset allocation, annuity allocation, and dynamic withdrawal strategy individually, we instead must determine the relative impact of changing each assumption within the test generator. Given the fact that there are three variables with two possible usage types ("yes" or "no") there are eight total different simulations to consider to estimate Gamma. These combinations, along with the resulting amount of certainty-equivalent utility-adjusted income, Y, for the test, are included in Table 2.

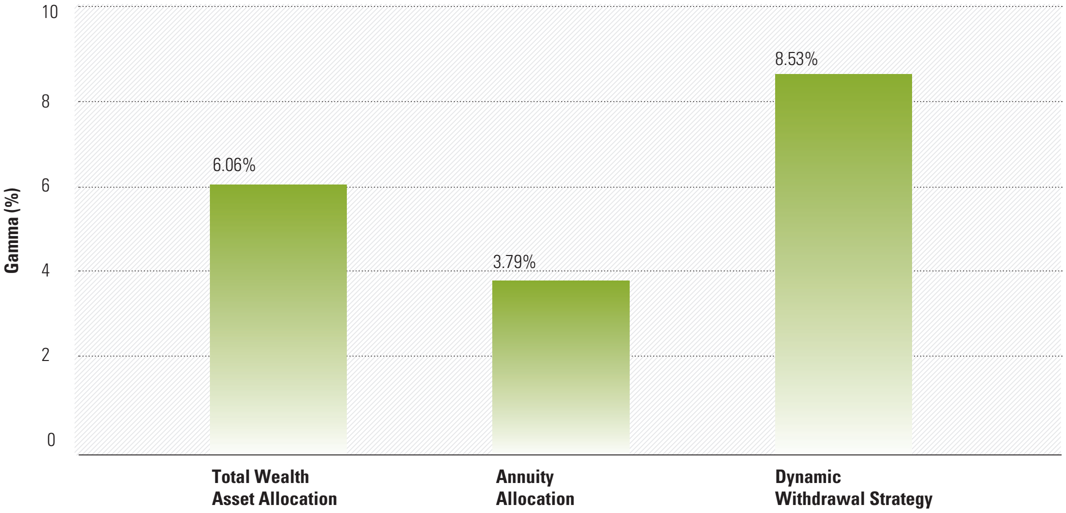
Table 2: Test 1 Combinations

Test Scenario	Total Wealth Asset Allocation	Annuity Allocation	Dynamic Withdrawal Strategy	*Y*
1	No	No	No	63.77
2	Yes	No	No	63.92
3	No	Yes	No	65.79
4	Yes	Yes	No	65.90
5	No	No	Yes	65.28
6	Yes	No	Yes	74.56
7	No	Yes	Yes	67.12
8	Yes	Yes	Yes	75.32

Given these values, it is possible to compare the differences between the respective pairs to determine the aggregate impact. For example, in order to determine the potential benefit of total wealth asset allocation we would just subtract the Y values for the following pairs: 2 & 1, 4 & 3, 6 & 5, and 8 & 7, respectively. These differences then tell us the relative improvement in income for each of three different potential decision combinations. The aggregate increase in income from the simplest scenario 1 to the most advanced scenario 8 is 18.1%.

Given an aggregate increase of 18.1% from the Gamma-optimized approach we can determine the individual contributions of total wealth asset allocation, annuity allocation, and dynamic withdrawal strategy by their weighted-average pair differences. We present these results in Figure 3. Among the three types, a dynamic withdrawal strategy added the most Gamma, at 8.53%, versus total wealth asset allocation and annuity allocation at 6.05% and 3.79%, respectively.

Figure 3: Gamma Values for the First Test



Test 2: Tax Efficiency

For this analysis we created a simulator that contained two account types: a 401(k) account and a taxable account. For the 401(k) account, gains in the account are not realized until income is withdrawn from the account. All 401(k) income is assumed to be taxed at a 30% tax rate (which is slightly lower than the highest current marginal tax rate of 35%). For the taxable account, tax is due for all gains that are realized. We assume that all bond returns are realized annually and taxed at the 30% tax rate. We assume that stock returns come from 50% long-term capital gains (or qualified dividends), which are taxed at 15%, and from 50% short-term capital gains (or non-qualified dividends), which are taxed at 30%. Within the taxable account, 40% of all gains during the year are assumed to be realized by the investor, which is a relatively tax-efficient portfolio. We assume that all income withdrawals from the taxable account are sourced from “basis” first if the annual gains are not enough to support the distribution. We assume that the beginning basis is 80% of the taxable account value upon retirement. We assume that the account balances in the 401(k) account and the taxable account are equal. The key difference in this analysis is the location of the stocks and bonds and the sequence of the withdrawals from the two accounts. We assume that the equity allocation of the portfolio is a constant 40%, and this 40% equity allocation is maintained over the life of the portfolio. Stocks and bonds are first purchased in the 401(k) account in order to achieve the 40% equity target and then purchased in the taxable account, if necessary. In some cases this means realizing gains in order to maintain the target equity allocation. The key assumption, therefore, is that maintaining the target equity allocation is more important than tax efficiency. Also, if a consistent equity allocation were not maintained, the risk and return attributes of the portfolio could change considerably over the life of a given simulation, which would materially affect the results of the simulation.

We consider a total of nine different scenarios, three different asset location scenarios and three different withdrawal sequencing scenarios. Among the possible outcomes are an efficient scenario, a “split” scenario, and an inefficient scenario. The efficient scenario represents the most efficient possible solution for the given test, which is either allocating as much bonds as possible in the 401(k) account for the asset location test or withdrawing from the taxable account first for the sequencing test. The split scenario assumes everything is divided evenly among the options. The inefficient scenario represents the least-efficient possible solution, which is either holding as much stocks as possible in the 401(k) account for the asset location test or withdrawing from the 401(k) account first for the sequencing test. The scenario where both options are split (i.e., the double-split scenario) is assumed to be the “base scenario” and subsequent results from the other eight scenarios are compared against the results of the double-split scenario.

We based the analysis on a 5,000-trial Monte Carlo simulation in which we assumed that annual returns (and inflation) vary by year, but are identical across each of the nine scenarios. In this way the only differences that would result from a given simulation would be based on the overall tax efficiency of the portfolio. The annual returns and standard deviations for stocks and bonds are based on US Large-Cap Stocks and US Bonds assumptions in the Appendix. We also model inflation using the multivariate non-normal distribution. We assume that the cash flow retirement needs increase at the simulated rate of inflation.

In order to ensure the portfolio is drawn down to zero over the lifetime simulation (which is assumed to last no more than 100 years), we assume a 5% initial withdrawal rate. This is a slightly higher withdrawal rate than the first test, but we increased it to ensure the ending value of the base scenario (the double-split scenario) reaches $0 during the simulation. We calculated two different metrics. The first is the difference in total income generated by the portfolios. The second is the return difference associated with the decrease in income. This is a nonlinear optimization approach and provides the reader with an alpha-like metric when it comes to the importance of tax-efficient retirement income investing. We present the results of the two tests in Table 3.

As Table 3 shows, there is a significant cost associated with inefficient investing during retirement. This cost can be attributed to the "return drag" associated with paying taxes versus delaying payment. The difference in retirement income for the least efficient of the nine scenarios (inefficient asset location and 401(k) withdrawals first) to the most efficient (efficient asset location and taxable withdrawals first) is 19.09%. This is relatively significant difference, but it is important to point out that this is a comparison of the worst possible outcome to the best. The double-split scenario is likely a better proxy, because as opposed to assuming the investor is being actively unintelligent (i.e., investing in an inefficient portfolio, we assume the investor is unsure what to do and therefore spreads the portfolio and income across the available options.

Table 3: Asset Location and Withdrawal Sequencing (Income Order) Results

		Additional Income Generated **Asset Location Portfolio Efficiency**		
		Efficient	1/n	Inefficient
Income Order	401k First	0.71%	-4.06%	-10.86%
	Split	3.83%	0.00%	-3.75%
	Taxable First	8.23%	6.82%	4.95%

		Equivalent Return Impact **Asset Location Portfolio Efficiency**		
		Efficient	1/n	Inefficient
Income Order	401k First	0.07%	-0.24%	-0.78%
	Split	0.21%	0.00%	-0.25%
	Taxable First	0.43%	0.36%	0.25%

While this analysis included two common tax investment account types: 401(k) (or a Traditional IRA) and a taxable account, it did not include a Roth IRA account. A Roth account is excluded since most investors will not have significant assets in this account type; however, given the increasing flexibility of Traditional to Roth IRA rollovers, Roth IRAs are likely to become increasingly common account types for retirees. Roth IRA accounts are perhaps the most efficient for retirement income because there are no minimum required distributions, Roth IRA income does not affect Social Security benefit taxation, and Roth IRAs are very efficient from an estate tax planning perspective. Therefore, additional potential Gamma gains are possible for a retiree who has money in Roth IRA-type account.

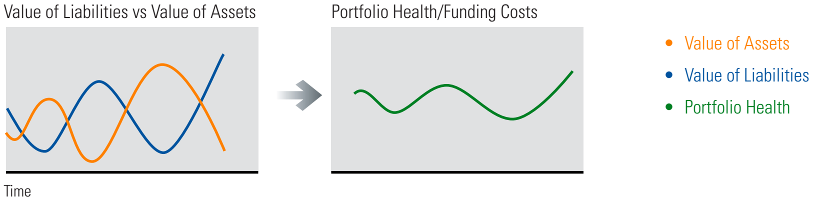
Liability-Relative Investing

The investment management process has traditionally focused on finding the optimal asset allocation that maximizes the expected return of a portfolio for a given level of risk. This asset-centric approach, however, is not really optimal because it ignores the fact that the purpose of the portfolio is to pay for an ongoing liability, which in the case of a retiree is to provide retirement income. In this instance, the appropriate or total portfolio should be optimized to include both assets and liabilities. Liability-focused methodologies are becoming increasingly popular for defined benefit plans, where there is a specific, though somewhat uncertain, future legal liability as well as for the design of defined contribution strategies in which there is a "soft" liability (see Idzorek [2008]).

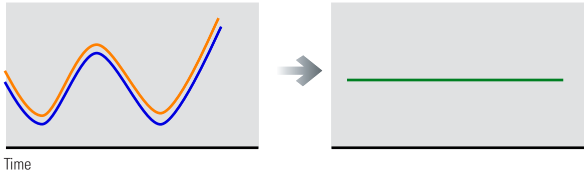
We depict the theoretical advantage of liability-relative optimization over an asset-only optimization framework in Figure 4. The top panel represents the asset-only approach and the bottom panel represents the liability-relative approach. On the left side of both panels, the blue line representing the evolving value of the liability is identical. In the top left graph, we see that an asset-only approach leads to a portfolio of assets with a value that may not always be moving in the same direction as the value of liabilities because the portfolio of assets is determined in isolation. This in turn leads to a portfolio whose health (and / or the cost associated with funding the portfolio) can vary significantly over time. In contrast, in the bottom left graph, we see that the liability-relative approach can lead to a portfolio of assets with a value that should move in sync with the value of the liabilities because the portfolio of assets is determined in the presence of the liability. This in turn leads to a portfolio whose health (and / or the cost associated with funding the portfolio) is steadier over time.

Figure 4: Improving Total Portfolio Health

Asset-only Approach



Liability-relative Approach



In order to determine the potential benefit of liability-relative optimized portfolios, we construct three different retirement portfolios each with a target return of the 6%. Note, this 6% return is the actual geometric return, or compounded return, experienced by the portfolio within the Monte Carlo simulation. The objective function for the liability relative optimization is :

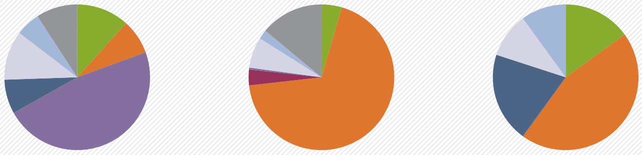
$$max(U_s) = E_s - \lambda \sigma_s^2 \qquad [5]$$

E_S = the expected value of assets less liabilities
σ_S = the standard deviation of assets less liabilities
λ = the risk aversion parameter

(See Waring [2004] for additional information on liability-relative optimization.) The liability in this model is inflation. This liability model is based on the assumption that a retiree's income goal is only linked to inflation. We assume a risk aversion parameter (λ) of 0.55. Note, our definition of inflation is effectively the CPI, although the actual inflation definition may vary for each retiree if based on forecasted health expenses, lifestyle expenses, etc.

The second portfolio is a traditional asset-only optimization. This approach is essentially the traditional Markowitz asset-only mean-variance optimization process. The final portfolio is a "naïve" portfolio and is the same naïve asset allocation used for Test 1. Figure 5 shows the allocations for the three portfolios.

Figure 5: Test Portfolios



		Liability-Relative Optimization	Asset-Only Optimization	Naïve Optimization
🟦	US Large Cap Stock	7.6	0.0	20.0
⬜	US Small Cap Stock	10.9	6.8	10.0
🟦	Non US Large Cap Stock	5.4	2.1	10.0
⬛	Emerging Markets Stock	9.2	14.0	0.0
🟧	US Bond	7.7	68.7	45.0
🟥	Non US Bond	0.0	3.5	0.0
🟪	US Tips	47.4	0.4	0.0
🟩	Cash	11.7	4.5	15.0

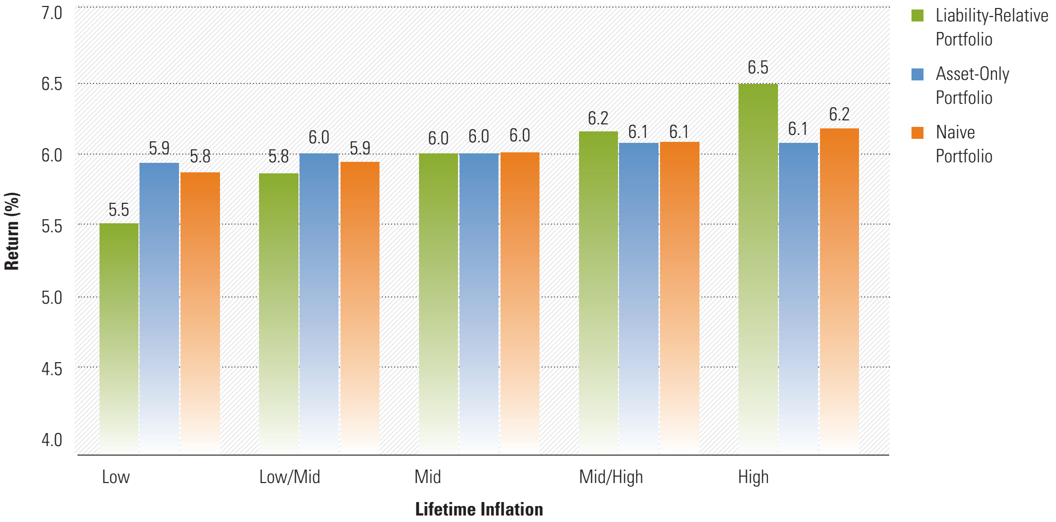
There are clearly significant differences among the three portfolios, most notably with the liability-relative optimized portfolio allocating heavily to US TIPS while the asset-only optimized portfolio and the naïve portfolio have large allocations to US Bonds. When viewed within an asset-only space (i.e., ignoring the liability), as shown in Table 4, the asset-only portfolio is the most efficient since all the portfolioshave the same expected return, but the asset-only portfolio has the lowest standard deviation. To test this from a liability-relative perspective, we conduct a liability-relative optimization, which is a special case where an asset (inflation) is held short. Through this lens, the liability-relative optimized portfolio is clearly the most efficient, because it has the highest return and lowest risk among the three options. In both an asset-only and liability-relative space, the naïve portfolio is the least efficient.

Table 4: Return and Standard Deviation Attributes of the Three Portfolios

	Liability-Relative Optimization	Asset-Only Optimization	Naïve Portfolio
Geometric Return	6.00%	6.00%	6.00%
Standard Deviation	7.45%	6.71%	8.44%
Surplus Geometric Return	3.74%	3.66%	3.63%
Surplus Standard Deviation	6.79%	7.38%	8.71%

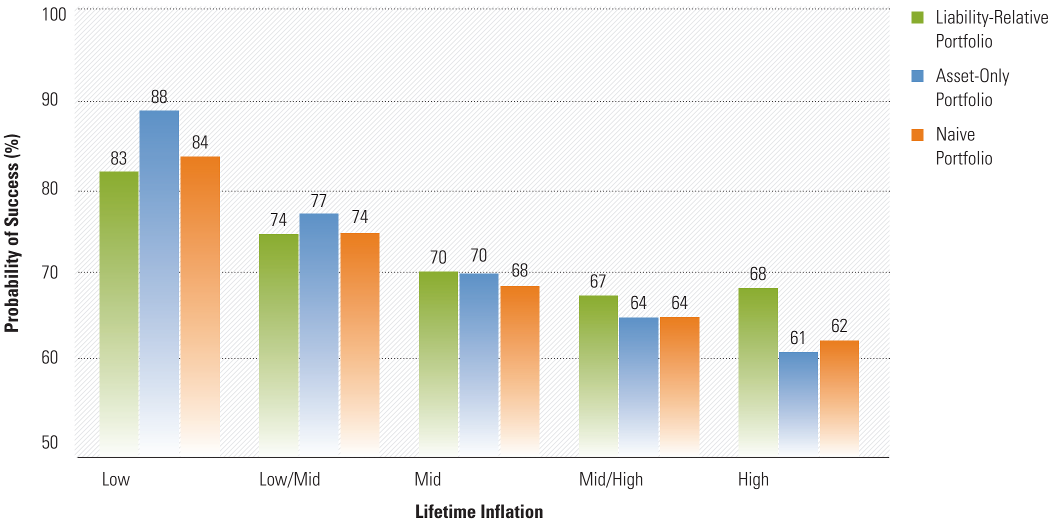
In order to understand how the three different portfolios performed in different interest rate environments, we ran a Monte Carlo simulation (5,000 trials) and divided the results into quintiles (groups of 1,000) based on the average annual inflation during the retirement distribution period. This allows us to compare how the portfolios performed in different inflationary environments. Figure 6 shows the median return for the three portfolios across the five different inflationary environments. Note how the liability-relative portfolio exhibits the highest slope at varying levels of inflation. When inflation is lower (and therefore the expected liability is lower) the liability-relative portfolio tends to have the lowest returns. In contrast, when inflation is highest the liability-relative portfolios have the highest returns. This allows for the asset returns to more effectively "match" the liability.

Figure 6: Average Annual Returns for the Inflation Quintiles for the Three Portfolios



This concept can also be demonstrated when looking at the probabilities of success for the portfolios in the different simulations, as shown in Figure 7. Each of the three portfolios does relatively well in the low inflation environment, but the liability-relative portfolio does especially well compared to the asset-only and the naive portfolio in the high-inflation environment.

Figure 7: Probabilities of Achieving the Target Withdrawal for the Inflation Quintiles for the Three Portfolios



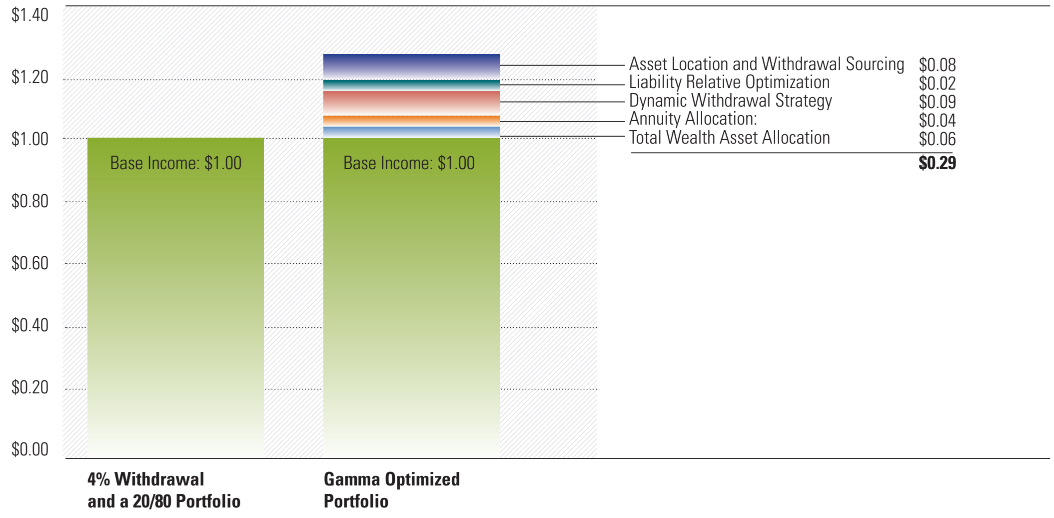
We use the constant relative risk-aversion utility function to measure the certainty-equivalent, or utility-adjusted, difference in retirement income for the liability-relative portfolio simulations and the asset-only and naïve portfolios (see equation [3]). Within the equation we substitute II with the total income goal replaced during retirement. This value is calculated by dividing the net present value of all retirement income achieved over the lifetime of the retirees plus the present value of the balance of assets at death by the net present value of the total income need. Assuming a risk tolerance parameter (θ) value of 0.25, we find that the liability-relative portfolio creates 0.84% more income on average, on certainty-equivalent basis, than the asset-only optimized portfolio, and 3.63% more income than the naïve portfolio.

If we assume the average investor has a portfolio that is somewhere between an asset-only optimized portfolio and a naïve portfolio (i.e., the average), we can say the average increase in certainty-equivalent income for liability-relative optimization is 2.23% versus a portfolio with an identical geometric return. Note, though, the certainty-equivalent income for the liability-relative portfolio is considerably higher during periods of high inflation, at 5.91% and 7.03%, respectively. However, since we are concerned with average events (not worst-case scenarios) the +2.23% is likely a better proxy for the expected benefit of liability-relative optimization.

Putting it All Together

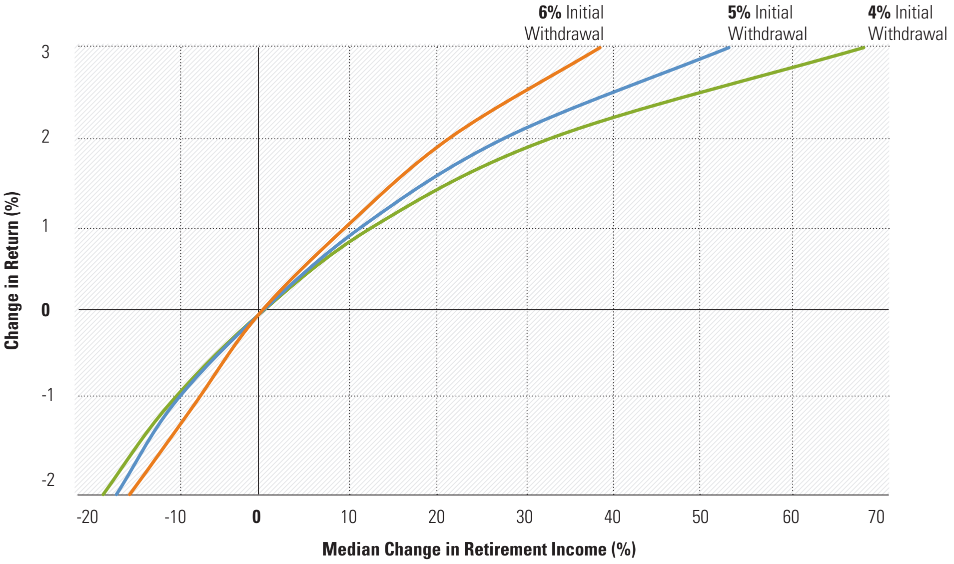
Up to this point we have conducted three different tests to determine the relative impact of five different kinds of Gamma: using total wealth to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products, tax-efficient allocation decisions, and liability-relative portfolio optimization. While there may be slight differences in some of the assumptions used within the tests, the results of each of the tests should add value independently of the other four (as was demonstrated in the first test). If we add the results from the five different types of Gamma tested, we find a Gamma of 28.8%, i.e., $1.29 for every $1 generated by the base set of assumptions. We display this concept visually in Figure 8, which shows the incremental and total income generated by each of the Gamma tests.

Figure 8: More Retirement Income with Gamma-Optimized Portfolios



An increase in certainty-equivalent utility-adjusted income of 28.8% is an impressive improvement in potential retirement income, but how does it relate from a traditional alpha perspective? In order to determine how much additional annual return, or alpha, is equivalent to the 28.8% Gamma, we conduct an additional analysis. We determine the total income generated for three different initial withdrawal rates (4%, 5%, 6%) and compare it to the income generated for portfolios with returns that are either higher or lower than the base portfolio by -2%, -1%, 0% (no change), +1%, +2%, and +3%. We compare the difference in the amount of income generated against the income generated by the 0% change (i.e., no change) portfolio. We show these results in Figure 9.

Figure 9: Relationship Between Additional Retirement Income and Changes in Returns for Different Initial Withdrawal Rates



By fitting a third-order polynomial to the curve depicting the 4% initial withdrawal, we estimate the equivalent annual return impact of a +28.8% increase in retirement income to be 1.82%. Table 5 shows how we attribute this Gamma-equivalent alpha among the five Gamma factors. This is likely to be significantly higher than any type of portfolio "alpha" that a financial advisor would be able to generate through fund selection or market timing. Also, while traditional portfolio alpha is a negative-sum game (since everyone cannot, on average, outperform the market), our results show that Gamma is not a zero-sum game and can be achieved by any investor who takes a smarter approach to generating retirement income.

Table 5: Additional Income Amounts and Gamma-Equivalent Alpha Values

Gamma Type	Additional Income Generated	Gamma Equivalent Alpha
Total Wealth Asset Allocation	6.1%	0.38%
Annuity Allocation	3.8%	0.24%
Dynamic Withdrawal Strategy	8.5%	0.54%
Liability Relative Optimization	2.2%	0.14%
Asset Location and Withdrawal Sourcing	8.2%	0.52%
Total	**28.8%**	**1.82%**

Conclusion

In this article, we introduce a new concept called "Gamma." We define Gamma as the additional value achieved by an individual investor from making more intelligent financial planning decisions. While Gamma varies for different types of investors, in this article we focus on five types of Gamma relevant to retirees: using a total wealth framework to determine the optimal asset allocation, a dynamic withdrawal strategy, incorporating guaranteed income products, tax-efficient allocation decisions, and liability-relative portfolio optimization. Among the five types of Gamma tested, using a dynamic withdrawal strategy was determined to be the most important, followed by making tax-efficient allocation decisions.In the aggregate, we estimate a retiree can be expected to generate 29% more income on a certainty-equivalent utility-adjusted basis utilizing a Gamma-efficient retirement income strategy when compared to our base scenario, of a 4% withdrawal rate and a 20% equity allocation portfolio. This additional income is equivalent to an average annual return increase of +1.82% (i.e., Gamma-equivalent alpha), which represents a significant improvement in portfolio efficiency for a retiree. Unlike traditional alpha, which is a zero-sum game and likely a negative-sum game after fees, we find that Gamma (and Gamma-equivalent alpha) can be achieved by anyone following an efficient financial planning strategy.

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Appendix

Nominal Market Assumptions[3]

Asset Class	Index	Returns	Std Dev	Kurtosis	Skewness
Cash	IA SBBI US 30 Day TBill TR USD	1.92%	3.18%	0.46	0.45
US Bonds	BarCap US Agg Bond TR USD	4.05%	6.51%	2.60	1.14
Non-US Bonds	IA Global ex-US Bond Composite	4.06%	10.56%	0.08	0.69
US TIPSs	BarCap Gbl Infl Linked US TIPS TR USD	3.57%	7.03%	-0.28	0.45
US Large-Cap Stocks	IA SBBI S&P 500 TR USD	9.61%	19.50%	0.02	-0.70
US Small-Cap Stocks	Russell 2000 TR USD	11.77%	24.68%	-0.33	-0.26
Non US Large Cap Stocks	MSCI EAFE GR USD	10.29%	21.05%	0.42	0.06
Emerging Markets Stocks	IA Emerging Markets Composite	15.17%	31.52%	-0.70	0.11
Inflation	IA SBBI US Inflation	2.23%	3.13%	1.65	1.48

Correlations	Cash	US Bonds	Non-US Bonds	US TIPSs	US Large Stocks	US Small Stocks	Non-US Stocks	Emerging Markets	Inflation
Cash	1.00	0.27	-0.13	-0.27	0.20	0.11	0.08	-0.20	0.66
US Bonds	0.27	1.00	0.18	0.51	0.28	0.09	0.01	-0.25	-0.21
Non-US Bonds	-0.13	0.18	1.00	0.35	0.06	-0.07	0.46	0.17	-0.08
US TIPSs	-0.27	0.51	0.35	1.00	-0.13	0.01	-0.06	0.07	0.52
US Large-Cap Stocks	0.20	0.28	0.06	-0.13	1.00	0.66	0.60	0.38	0.09
US Small-Cap Stocks	0.11	0.09	-0.07	0.01	0.66	1.00	0.42	0.42	0.26
Non-US Large-Cap Stocks	0.08	0.01	0.46	-0.06	0.60	0.42	1.00	0.62	0.06
Emerging Markets	-0.20	-0.25	0.17	0.07	0.38	0.42	0.62	1.00	0.10
Inflation	0.66	-0.21	-0.08	0.52	0.09	0.26	0.06	0.10	1.00

Gamma Type	Additional Income Generated	Gamma Equivalent Alpha
Total Wealth Asset Allocation	6.1%	0.38%
Annuity Allocation	3.8%	0.24%
Dynamic Withdrawal Strategy	8.5%	0.54%
Liability Relative Optimization	2.2%	0.14%
Asset Location and Withdrawal Sourcing	8.2%	0.52%
Total	**28.8%**	**1.82%**

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[3] The reader may note the assumed level of annual inflation (2.23%) is higher than the assumed return on cash (1.92%). Therefore, the authors are forecasting a negative real (inflation-adjusted) return on cash for this paper. These forecasts are based on Ibbotson's Capital Market Assumptions as of March 30, 2012. While this assumption may seem questionable, it is certainly valid given the current cash returns of effectively 0%.

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David Blanchett, CFA is head of retirement research for the Morningstar Investment Management division, which provides investment consulting, retirement advice, and investment management operations around the world. In this role, he works closely with the division's business leaders to provide research support for the group's consulting activities and conducts client-specific research primarily in the areas of financial planning, tax planning, and annuities. He is responsible for developing new methodologies related to strategic and dynamic asset allocation, simulations based on wealth forecasting, and other investment and financial planning areas for the investment consulting group, and he also serves as chairman of the advice methodologies investment subcommittee.

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Kaplan led the development of the quantitative methodologies behind the revised versions of Morningstar RatingTM for funds (Morningstar's 'Star' rating), the Morningstar Style BoxTM, and the Morningstar family of indexes.
Previously, Kaplan has served as quantitative research director for Morningstar Europe in London, director of quantitative research in the United States, and chief investment officer of Morningstar Associates, LLC, a registered investment advisor and wholly owned subsidiary of Morningstar, Inc., where he developed and managed the investment methodology for Morningstar's retirement planning and advice services.

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The results from the simulations described within are hypothetical in nature and not actual investment results or guarantees of future results.

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